p, q are distinct odd primes.Consider a modified RSA scheme where we define:
$\lambda(n) = \frac{(p-1)(q-1)}{gcd(p-1,q-1)}$ and the congruence we must satisfy is: $ab \equiv 1 mod \lambda(n)$

recall the definition of encoding/decoding functions:
$e_k(x) = x^b (mod n)$ and $d_k(y) = y^a (mod n)$
we want to show that $d_k(e_k(x)) = x$
I'm assuming $x \in Z_n^*$ after some simplification we find we must show that:
$x^{ab} \equiv x (mod n)$

since n=pq where p,q distinct prime, the chinese remainder theorem allows us prove for p,q, individually:
$x^{ab} \equiv x (mod p)$ and $x^{ab} \equiv x (mod q)$ we will consider the case for mod p

since we know, for some positive integer k that: $ab = k \cdot \lambda(n) + 1$
we can reduce this to $x^{ab} \equiv (x^{\lambda(n)})^k x (mod n)$

next we expand: $\lambda(n) = \frac{pq-q-p+1}{gcd(p-1,q-1)}$ plugging this in:
$x^{ab} \equiv (x^{\frac{kpq}{gcd(p-1,q-1)}})(x^{\frac{-kq}{gcd(p-1,q-1)}})(x^{\frac{-kp}{gcd(p-1,q-1)}})(x^{\frac{k}{gcd(p-1,q-1)}})x (mod p)$
re-arranging this:
$x^{ab} \equiv (x^p)^{\frac{kq}{gcd(p-1,q-1)}} (x)^{\frac{-kq}{gcd(p-1,q-1)}} (x^p)^{\frac{-k}{gcd(p-1,q-1)}} (x)^{\frac{k}{gcd(p-1,q-1)}} x (mod p)$
Now since p is prime, we can use fermats little theorem that says for any integer t: $t^p \equiv t (mod p)$ Thus we have:

$x^{ab} \equiv (x)^{\frac{kq}{gcd(p-1,q-1)}} (x)^{\frac{-kq}{gcd(p-1,q-1)}} (x)^{\frac{-k}{gcd(p-1,q-1)}} (x)^{\frac{k}{gcd(p-1,q-1)}} x (mod p)$
Cancelling the terms we have then:
$x^{ab} \equiv x (mod p)$

The proof is the same for mod q, the combination of these two with the CRT prove the overall problem.